Q.8	A box contains five balls of same size and shape. Three of them are green coloured balls and two of them are orange coloured balls. Balls are drawn from the box one at a time. If a green ball is drawn, it is not replaced. If an orange ball is drawn, it is replaced with another orange ball. First ball is drawn. What is the probability of getting an orange ball in the next draw?
(A)	$\dfrac{1}{2}$
(B)	$\dfrac{8}{25}$
(C)	$\dfrac{19}{50}$
(D)	$\dfrac{23}{50}$

ANSWER GIVEN as per official GATE key – **OPTION D (23/50)**

There is a slight ambiguity in the interpretation.

One of the interpretations is like –

When a green ball is drawn *it is not replaced* means a ball is drawn ,if it is green it is **not put back in the bag.**

If an orange ball is drawn it is **replaced with another** orange ball.

Here "replaced with another " orange ball means another orange ball along with the orange ball drawn is put back in the bag. (2 orange balls added)

The question means to say that

If you draw a green ball, then don't put back. If you draw the orange ball, then put *two* orange balls back.

Or in another way

If it is green, it is not replaced (put back) into the urn. Otherwise it is replaced *alongside* another ball of same color.

There is an interpretation ambiguity in this question.

POSSIBLE SOLUTIONS TO THE GATE QUESTION-

P(orange ball in the second draw) = *P(Orange ball in 2nd draw/green ball in 1st draw) * P(green ball in 1st draw) + P(Orange ball in 2nd draw/orange ball in 1st draw) * P(orange ball in 1st draw)*

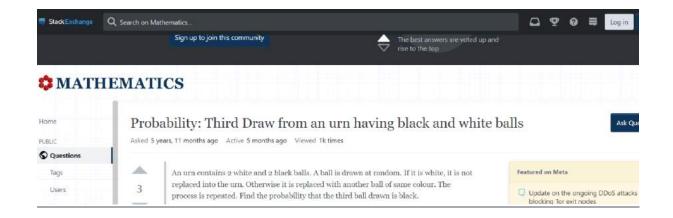*Assumption 1 -Green ball drawn is not replaced.* If you draw the orange ball, then put *two* orange balls back.

$2/4 * 3/5 + 3/6 * 2/5 = ½$ *(option A)*

Assumption 2 – Green ball drawn is not replaced and Orange ball is drawn with replacement.
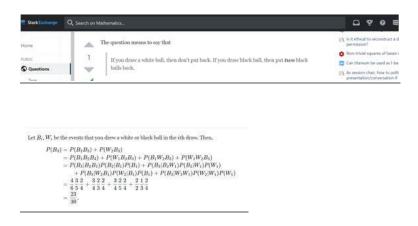
$2/4 * 3/5 + 2/5 * 2/5 = 23/50$ *(option D)*

One similar ques in StackExchange

Link - https://math.stackexchange.com/questions/1695378/probability-third-draw-from-an-urn-having-black-and-white-balls

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Probability: Third Draw from an urn having black and white balls

Asked 5 years, 11 months ago   Active 5 months ago   Viewed 1k times

3

An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise it is replaced with another ball of same colour. The process is repeated. Find the probability that the third ball drawn is black.

Featured on Meta

Update on the ongoing DDoS attacks blocking Tor exit nodes

StackExchange Solution –

The question means to say that

1

If you draw a white ball, then don't put back. If you draw black ball, then put *two* black balls back.

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Let B_i, W_i be the events that you drew a white or black ball in the ith draw. Then,

$$P(B_3) = P(B_2B_3) + P(W_2B_3)$$
$$= P(B_1B_2B_3) + P(W_1B_2B_3) + P(B_1W_2B_3) + P(W_1W_2B_3)$$
$$= P(B_3|B_2B_1)P(B_2|B_1)P(B_1) + P(B_3|B_2W_1)P(B_2|W_1)P(W_1)$$
$$+ P(B_3|W_2B_1)P(W_2|B_1)P(B_1) + P(B_3|W_2W_1)P(W_2|W_1)P(W_1)$$
$$= \frac{4}{6}\frac{3}{5}\frac{2}{4} + \frac{3}{4}\frac{2}{3}\frac{2}{4} + \frac{3}{4}\frac{2}{5}\frac{2}{4} + \frac{2}{2}\frac{1}{3}\frac{2}{4}$$
$$= \frac{23}{30}.$$

Similar question has been asked in ISC 2015 paper

Link- 1.
https://books.google.co.in/books?id=tfJVEAAAQBAJ&pg=PA97&lpg=PA97&dq=%22+is+replaced+with+another%22+probability+ball&source=bl&ots=DAdAdfdJ8O&sig=ACfU3U1AMIklCpOEeEKAJUVFr-wIaxlYSw&hl=en&sa=X&ved=2ahUKEwiesL6Lr5L2AhW-S2wGHWOWAho4HhDoAXoECAcQAg#v=onepage&q&f=false

OSWAL BOOK page – 97- QUES 6

 Ques -8

An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.   [ISC, 2015]

SOLUTION GIVEN BY OSWAL AS PER ISC 2015 MARKING SCHEME

Sol. Case I : Here, it is assumed that, the black ball drawn is replaced along with another ball of the same colour.

$P(E) = P(WWB) + P(WBB) + P(BWB) + P(BBB)$

$$= \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} + \frac{2}{4} \times \frac{2}{3} \times \frac{3}{4} + \frac{2}{4} \times \frac{2}{5} \times \frac{3}{4}$$

$$+ \frac{2}{4} \times \frac{3}{5} \times \frac{4}{6}$$

$$= \frac{1}{6} + \frac{1}{4} + \frac{3}{20} + \frac{1}{5}$$

$$= \frac{10 + 15 + 9 + 12}{60} = \frac{46}{60} = \frac{23}{30}$$

Case II :

Here, it is assumed that, the black ball drawn is with replacement.

$P(E) = P(WWB) + P(WBB) + P(BWB) + P(BBB)$

$$= \left(\frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} \right) + \left(\frac{2}{4} \times \frac{2}{3} \times \frac{2}{3} \right)$$

$$+ \left(\frac{2}{4} \times \frac{2}{4} \times \frac{2}{3} \right) + \left(\frac{2}{4} \times \frac{2}{4} \times \frac{2}{4} \right)$$

$$= \frac{1}{6} + \frac{2}{9} + \frac{1}{6} + \frac{1}{8} = \frac{49}{72}$$

[ISC Marking Scheme, 2015]

ANOTHER SOLUTION OF THE SAME ISC 2015 QUES

(a) Please note: The number of balls in the urn change based on what color is drawn.

Every time you get a Black ball, not only the ball is put back in the urn, another black ball is put in the urn. So the number of balls increase every time you pull a black ball.

In case you draw a white ball, it is not put back in the urn. So the number of balls decrease when you pull white ball.

Therefore the probability that the third ball is black is:

$$P(E) = P(WWB) + P(WBB) + P(BWB) + P(BBB)$$

$$= \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} + \frac{2}{4} \times \frac{2}{3} \times \frac{3}{4} + \frac{2}{4} \times \frac{2}{5} \times \frac{3}{4} + \frac{2}{4} \times \frac{3}{5} \times \frac{4}{6}$$

$$= \frac{1}{6} + \frac{1}{4} + \frac{3}{20} + \frac{1}{5}$$

$$= \frac{23}{30}$$

A similar QUES from ISC_XII_MATHEMATICS MODEL PAPER

Link - https://drive.google.com/file/d/1xQoNCCfXn-tR6HckwFpooy6ttybGXfOt/view   page -3-- Ques- 8

An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.

The third ball will be black in these following cases

W-W-B

W-B-B

B-W-B

B-B-B These are the all possible way.

Required probability is = P(WWB)+P(WBB)+P(BWB)+P(BBB)

$$= \frac{2}{4}\frac{1}{3}1 + \frac{2}{4}\frac{2}{3}\frac{3}{4} + \frac{2}{4}\frac{2}{5}\frac{3}{4} + \frac{2}{4}\frac{3}{5}\frac{4}{6} = \frac{1}{6} + \frac{1}{4} + \frac{3}{20} + \frac{1}{5} = \frac{23}{30}$$

POSSIBLE INTERPRETATIONS TO THE ABOVE QUES-

As with many probability problems the answer might vary with the interpretation of the problem. The phrase " it is replaced with another ball " can have different interpretation viz., the black ball is placed in the urn along with another black ball ·.

Both option A (1/2) and option D (23/ 50) can be the possible answers depending on the way of interpreting the question.

REFERENCE

- https://math.stackexchange.com/questions/1695378/probability-third-draw-from-an-urn-having-black-and-white-balls -

  StackExchange

- https://www.shaalaa.com/question-bank-solutions/an-urn-contains-2-white-2-black-balls-ball-drawn-random-if-it-white-it-not-replaced-urn-otherwise-it-replaced-another-ball-same-colour-process-repeated-find-probability-that-third-ball-drawn-black-conditional-probability_19761

- https://drive.google.com/file/d/1xQoNCCfXn-tR6HckwFpooy6ttybGXfOt/view?usp=sharing --

  ISC XII MATHEMATICS MODEL PAPER – Page 3 -Ques 8

  https://icsecbsemath.com/2018/07/21/2015-isc-class-12-board-paper-solution-mathematics/   **Ques -8**

- https://books.google.co.in/books?id=tfJVEAAAQBAJ&pg=PA97&lpg=PA97&dq=%22+is+replaced+with+another%22+probability+ball&source=bl&ots=DAdAdfdJ8O&sig=ACfU3U1AMIklCpOEeEKAJUVFr-wIaxlYSw&hl=en&sa=X&ved=2ahUKEwiesL6Lr5L2AhW-S2wGHWOWAho4HhDoAXoECAcQAg#v=onepage&q&f=false

- https://www.toppr.com/ask/question/an-urn-contains-2-white-and-2-black-balls-a/