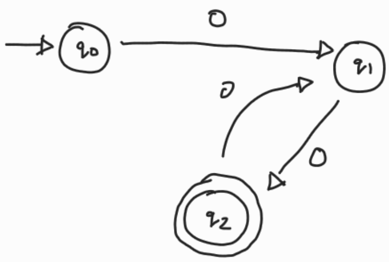
$S_1 : \{ 0^{2n} \mid n \geq 1 \}$ is a regular language.

Ans) Yes, the DFA is 

$S_2 \; \{ 0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1 \}$ is a regular language.

Ans) False, we can show that this language doesn't satisfy pumping lemma.

<u>Proof</u>

Suppose language is regular, then there exists a pumping length p such that for all strings $w \in \mathcal{L}$ with $|w| \geq p$,

w can be divided as $w = xyz$ and satisfy following conditions –

① $|y| > 0$

② $|xy| \leq p$

③ for all $i \geq 0$, $xy^i z \in \mathcal{L}$

Let $w = 0^p 1^p 0^{2p}$, then $|w| = 4p \geq p$ and $w \in \mathcal{L}$.

We now show that there is no way to divide w into x, y, z and satisfy ①, ② and ③.

Take $x = 0^\alpha$, $y = 0^\beta$, $z = 0^{p-\alpha-\beta} 1^p 0^{2p}$; where $\beta > 0$ and $\alpha + \beta \leq p$.

Then $xy^0 z = 0^\alpha 0^{p-\alpha-\beta} 1^p 0^{2p} = 0^{p-\beta} 1^p 0^{2p} \notin \mathcal{L}$, since $\beta > 0$

So with any appropriate value of α and β satisfying $\beta > 0$ and $\alpha + \beta \leq p$, we cannot pump down w.

Since $\mathcal{L}$ doesn't satisfy the pumping lemma, $\mathcal{L}$ is non-regular.