1. The area lying in the first quadrant and bounded by the circle

$$x^2 + y^2 = 4$$

and lines

$$x = 0 \text{ and } x = 1$$

is given by

(A) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$ (C) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (D) $\frac{\pi}{6} + \frac{\sqrt{3}}{2}$

2. If (x_1, y_1) and (x_2, y_2) are the opposite endpoints of a diameter of a circle, then the equation of the circle is given by

(A) $(x - x_1)(y - y_1) + (x - x_2)(y - y_2) = 0$

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3. If α, β and γ are the roots of the equation $x^3 + 3x^2 - 8x + 1 = 0$, then an equation whose roots are $\alpha + 1$, $\beta + 1$ and $\gamma + 1$ is given by

(A) $y^3 - 11y + 11 = 0$ (B) $y^3 - 11y - 11 = 0$

(C) $y^3 + 13y + 13 = 0$ (D) $y^3 + 6y^2 + y - 3 = 0$

4. Let $S \subseteq \mathbb{R}$. Consider the statement:

> "There exists a continuous function $f : S \to S$ such that $f(x) \neq x$ for all $x \in S$."

This statement is false if S equals

(A) $[2, 3]$ (B) $(2, 3]$ (C) $[-3, -2] \cup [2, 3]$ (D) $(-\infty, \infty)$

5. If A is a 2×2 matrix such that $\operatorname{trace} A = \det A = 3$, then what is the trace of A^{-1}?

(A) 1 (B) 1/3 (C) 1/6 (D) 1/2

6. In a class of 80 students, 40 are girls and 40 are boys. Also, exactly 50 students wear glasses. Then the set of all possible numbers of boys without glasses is

(A) $\{0, \ldots, 30\}$ (B) $\{10, \ldots, 30\}$ (C) $\{0, \ldots, 40\}$ (D) none of these

7. Let $n \geq 3$ be an integer. Then the statement

$$(n!)^{1/n} \leq \frac{n+1}{2}$$

is

(A) true for every $n \geq 3$

(B) true if and only if $n \geq 5$

(C) not true for $n \geq 10$

(D) true for even integers $n \geq 6$, not true for odd $n \geq 5$

8. Let X_1, X_2 and X_3 be chosen independently from the set $\{0, 1, 2, 3, 4\}$, each value being equally likely. What is the probability that the arithmetic mean of X_1, X_2 and X_3 is the same as their geometric mean?

(A) $\frac{1}{5^2}$ (B) $\frac{1}{5^3}$ (C) $\frac{3!}{5^3}$ (D) $\frac{3}{5^3}$

9. A function $y(x)$ that satisfies

$$\frac{dy}{dx} + 4xy = x$$

with the boundary condition $y(0) = 0$ is

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10. The inequality $|x^2 - 5x + 4| > (x^2 - 5x + 4)$ holds if and only if

(A) $1 < x < 4$ (B) $x \leq 1$ and $x \geq 4$

(C) $1 \leq x \leq 4$ (D) x takes any value except 1 and 4

11. The digit in the unit's place of the number 2017^{2017} is

(A) 1 (B) 3 (C) 7 (D) 9

12. Which of the following statements is true?

(A) There are three consecutive integers with sum 2015

(B) There are four consecutive integers with sum 2015

(C) There are five consecutive integers with sum 2015

(D) There are three consecutive integers with product 2015

13. An even function $f(x)$ has left derivative 5 at $x = 0$. Then

(A) the right derivative of $f(x)$ at $x = 0$ need not exist

(B) the right derivative of $f(x)$ at $x = 0$ exists and is equal to 5

(C) the right derivative of $f(x)$ at $x = 0$ exists and is equal to -5

(D) none of the above is necessarily true

14. Let (v_n) be a sequence defined by $v_1 = 1$ and

$$v_{n+1} = \sqrt{{v_n}^2 + \left(\frac{1}{5}\right)^n}$$

for $n \geq 1$. Then $\lim_{n\to\infty} v_n$ is

(A) $\sqrt{5/3}$ (B) $\sqrt{5/4}$ (C) 1 (D) nonexistent

15. The diagonal elements of a square matrix M are odd integers while the off-diagonals are even integers. Then

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(C) there is not enough information to decide the singularity of M

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16. Let (x_n) be a sequence of real numbers such that the subsequences (x_{2n}) and (x_{3n}) converge to limits K and L respectively. Then

(A) (x_n) always converges

(B) if $K = L$, then (x_n) converges

(C) (x_n) may not converge, but $K = L$

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17. Suppose that X is chosen uniformly from $\{1, 2, ..., 100\}$ and given $X = x$, Y is chosen uniformly from $\{1, 2, ..., x\}$. Then $P(Y = 30) =$

(A) $\frac{1}{100}$

(B) $\frac{1}{100} \times \left(\frac{1}{30} + \cdots + \frac{1}{100}\right)$

(C) $\frac{1}{30}$

(D) $\frac{1}{100} \times \left(\frac{1}{1} + \cdots + \frac{1}{30}\right)$

18. Consider the following system of equations:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ a & 9 & b & 10 \\ 6 & 8 & 10 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The locus of all $(a, b) \in \mathbb{R}^2$ such that this system has at least two distinct solutions for (x_1, x_2, x_3, x_4) is

(A) a parabola (B) a straight line (C) entire $\mathbb{R}^2$ (D) a point

19. If α, β and γ are the roots of $x^3 - px + q = 0$, then the value of the determinant

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$$

is

(A) p (B) p^2 (C) 0 (D) $p^2 + 6q$

20. The number of ordered pairs (X, Y), where X and Y are $n \times n$ real matrices such that $XY - YX = I$ is

(A) 0 (B) 1 (C) n (D) infinite

21. There are four machines and it is known that exactly two of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. The probability that only two tests are required is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

22. The five vowels—A, E, I, O, U—along with 15 X's are to be arranged in a row such that no X is at an extreme position. Also, between any two vowels there must be at least 3 X's. The number of ways in which this can be done is

(A) 1200 (B) 1800 (C) 2400 (D) 3000

23. What is the smallest degree of a polynomial with real coefficients and having roots 2ω, $2+3\omega$, $2\omega^2$, $-1-3\omega$ and $2-\omega-\omega^2$?

[Here $\omega \neq 1$ is a cube root of unity.]

(A) 5 (B) 7 (C) 9 (D) 10

24. The number of polynomial functions f of degree ≥ 1 satisfying

$$f(x^2) = (f(x))^2 = f(f(x))$$

for all real x, is

(A) 0 (B) 1 (C) 2 (D) infinitely many

25. For $a, b \in \mathbb{R}$, and $b > a$, the maximum possible value of the integral

$$\int_a^b (7x - x^2 - 10)dx$$

is

(A) $\frac{7}{2}$ (B) $\frac{9}{2}$ (C) $\frac{11}{2}$ (D) none of these

26. Let n be the number of ways in which 5 men and 7 women can stand in a queue such that all the women stand consecutively. Let m be the number of ways in which the same 12 persons can stand in a queue such that exactly 6 women stand consecutively. Then the value of $\frac{m}{n}$ is

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27. A box contains 5 fair coins and 5 biased coins. Each biased coin has probability of a head $\frac{4}{5}$. A coin is drawn at random from the box and tossed. Then a second coin is drawn at random from the box (without replacing the first one). Given that the first coin has shown head, the conditional probability that the second coin is fair, is

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28. Let H be a subgroup of a group G and let N be a normal subgroup of G. Choose the correct statement:

(A) $H \cap N$ is a normal subgroup of both H and N

(B) $H \cap N$ is a normal subgroup of H but not necessarily of N

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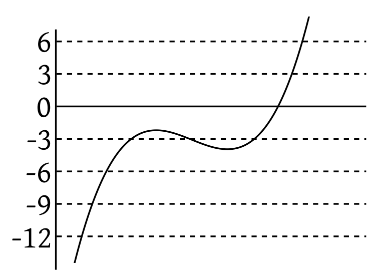
29. Suppose the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 3 \\ a & b & b & 1 \end{pmatrix}$$

is 2 for some real numbers a and b. Then b equals

(A) 1 (B) 3 (C) 1/2 (D) 1/3

30. The graph of a cubic polynomial $f(x)$ is shown below. If k is a constant such that $f(x) = k$ has three real solutions, which of the following could be a possible value of k?



(A) 3 (B) 0 (C) -7 (D) -3

2017

BOOKLET NO. TEST CODE : **MMA**

Forenoon

Questions : 30	**Time : 2 hours**

Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answersheet.

For each question, there are four suggested answers of which only one is correct. For each question indicate your choice of the correct answer by darkening the appropriate oval (⬬) completely on the answer sheet.

4 marks are allotted for each correct answer,
0 mark for each incorrect answer and
1 mark for each unattempted question.

ALL ROUGH WORK MUST BE DONE ON THIS BOOKLET ONLY.
YOU ARE NOT ALLOWED TO USE CALCULATORS IN ANY FORM.

STOP! WAIT FOR THE SIGNAL TO START.

1. If A is a 2×2 matrix such that $\operatorname{trace} A = \det A = 3$, then what is the trace of A^{-1}?

(A) 1 (B) $1/3$ (C) $1/6$ (D) $1/2$

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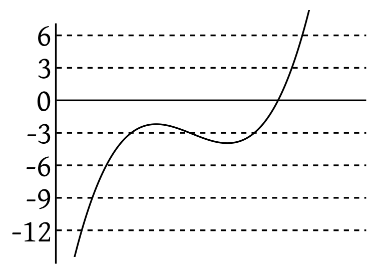

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