POST-GRADUATE DIPLOMA IN COMPUTER APPLICATIONS

TEST CODE : **DCG** (Objective Type) **2015**

SYLLABUS

Algebra - Arithmetic, Geometric and Harmonic Progression. Series and sequence. Permutation and combination. Binomial theorem. Theory of quadratic equations. Inequalities. Elementary set theory. Vectors and matrices. Determinant, rank and inverse of a matrix. Solution of linear equations.

Trigonometry - Basic concepts. Compound, multiple and submultiple angles. Inverse circular functions. Solution of trigonometric equations. Properties of triangles.

Coordinate geometry - Straight line, circle, parabola, ellipse and hyperbola.

Calculus - Taylor and Maclaurin series. Function, limit and continuity of a function of one real variable. Differentiation and integration of a function of one real variable with applications. Definite integrals. Areas using integrals. Maxima and minima and their applications.

SAMPLE QUESTIONS

<u>Note:</u> *For each question there are four suggested answers of which only one is correct.*

1. The sequence $\frac{1}{\log_3 2}, \frac{1}{\log_6 2}, \frac{1}{\log_{12} 2}, \frac{1}{\log_{24} 2} \cdots$ is in

   (A) Arithmetic progression (AP)
   (B) Geometric progression (GP)
   (C) Harmonic progression (HP)
   (D) None of these.

2. Let $S = \{6, 10, 7, 13, 5, 12, 8, 11, 9\}$, and $a = \sum_{x \in S} (x - 9)^2$ & $b = \sum_{x \in S} (x - 10)^2$. Then

   (A) $a < b$ (B) $a > b$ (C) $a = b$ (D) None of these.

3. The value of

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix}$$

   is

   (A) $abcd(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d})$ (B) $abcd(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d})$
   (C) $1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$ (D) None of these.

4. If $\tan\ x = p + 1$ and $\tan\ y = p - 1$, then the value of $2 \cot\ (x - y)$ is

   (A) $2p$ (B) p^2 (C) $(p + 1)(p - 1)$ (D) $\frac{2p}{p^2 - 1}$.

5. If

$$f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

then the value of $(f(x))^2$ is

(A) $f(x)$ (B) $f(2x)$ (C) $2f(x)$ (D) None of these.

6. The coefficient of x^2 in the product $(1+x)(1+2x)(1+3x)\cdots(1+10x)$ is

(A) 1320 (B) 1420 (C) 1120 (D) None of these.

7. Let $x^2 - 2(4k-1)x + 15k^2 - 2k - 7 > 0$ for any real value of x. Then the integer value of k is

(A) 2 (B) 4 (C) 3 (D) 1.

8. Let $S = \{0, 1, 2, \cdots, 25\}$ and $T = \{n \in S : n^2 + 3n + 2$ is divisible by 6 $\}$. Then the number of elements in the set T is

(A) 16 (B) 17 (C) 18 (D) 10.

9. Let a be the 81-digit number of which all the digits are equal to 1. Then the number a is,

(A) divisible by 9 but not divisible by 27
(B) divisible by 27 but not divisible by 81
(C) divisible by 81
(D) None of the above.

10. The 5000^{th} term of the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \cdots$ is

(A) 98 (B) 99 (C) 100 (D) 101.

11. Let two systems of linear equations be defined as follows:

$$P: \begin{array}{rl} x+y & = 1 \\ 3x+3y & = 3 \\ 5x+5y & = 5 \end{array} \qquad \text{and} \qquad Q: \begin{array}{rl} x+y & = 3 \\ 2x+2y & = 4 \\ 5x+5y & = 12 \end{array}.$$

Then,

(A) P and Q are inconsistent
(B) P and Q are consistent
(C) P is consistent but Q is inconsistent
(D) None of the above.

12. The highest power of 3 contained in $1000!$ is

(A) 198 (B) 891 (C) 498 (D) 292.

13. For all the natural number $n \geq 3$, $n^2 + 1$ is

(A) divisible by 3 (B) not divisible by 3
(C) divisible by 9 (D) None of these.

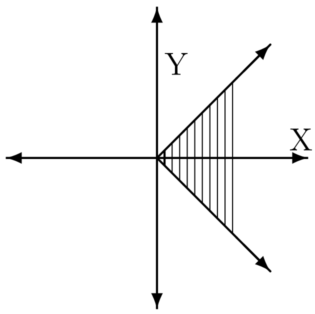
14. For natural numbers n, the inequality $2^n > 2n + 1$ is valid when

(A) $n \geq 3$ (B) $n < 3$ (C) $n = 3$ (D) None of these.

15. The smallest integer n for which $1 + 2 + 2^2 + 2^3 + 2^4 + \ldots + 2^n$ exceeds 9999, given that $\log_{10} 2 = 0.30103$, is

(A) 12 (B) 13 (C) 14 (D) None of these.

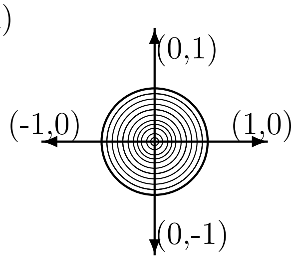
16. The shaded region in the following diagram represents the relation



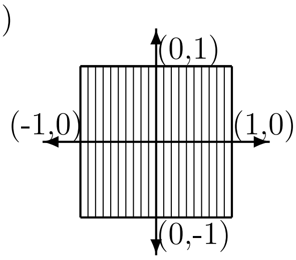
(A) $y \leq x$ (B) $|y| \leq |x|$ (C) $y \leq |x|$ (D) $|y| \leq x$.

17. The set $\{(x, y) : |x| + |y| \leq 1\}$ is represented by the shaded region in

(A)



(B)



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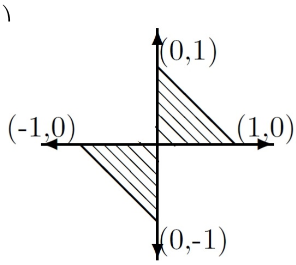
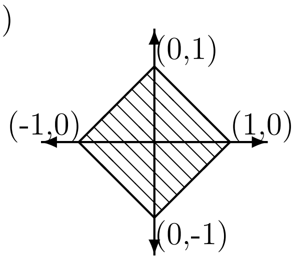


(D)



18. The value of $(1.1)^{10}$ correct to 4 decimal places is

(A) 2.4512 (B) 1.9547 (C) 2.5937 (D) 1.4512.

19. The expression $3^{2n+1} + 2^{n+2}$ is divisible by 7 for

(A) all positive integer values of n
(B) all non-negative integer values of n
(C) only even integer values of n
(D) only odd integer values of n.

20. The total number of factors of 3528 greater than 1 but less than 3528 is

(A) 35 (B) 36 (C) 34 (D) None of these.

21. The value of the term independent of x in the expansion of $(1-x)^2\left(x+\frac{1}{x}\right)^7$ is

(A) -70 (B) 70 (C) 35 (D) None of these.

22. The value of

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

is

(A) 0 (B) 1 (C) -1 (D) None of these.

23. The value of $\log_2 e - \log_4 e + \log_8 e - \log_{16} e + \log_{32} e - \cdots$ is

(A) -1 (B) 0 (C) 1 (D) None of these.

24. If the letters of the word COMPUTER be arranged in random order, the number of arrangements in which the three vowels O, U and E occur together is

(A) 8! (B) 6! (C) 3!6! (D) None of these.

25. If α and β be the roots of the equation $x^2 + 3x + 4 = 0$, then the eqation with roots $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is

(A) $x^2 + 2x + 63 = 0$ (B) $x^2 - 63x + 2 = 0$
(C) $x^2 - 2x - 63 = 0$ (D) none of the above

26. If r be the ratio of the roots of the equation $ax^2 + bx + c = 0$, then

(A) $\frac{r}{b} = \frac{r+1}{ac}$ (B) $\frac{r+1}{b} = \frac{r}{ac}$ (C) $\frac{(r+1)^2}{r} = \frac{b^2}{ac}$ (D) $(\frac{r}{b})^2 = \frac{r+1}{ac}$.

27. If A be the set of triangles in a plane and R^+ be the set of all positive real numbers, then the function $f : A \to R^+$, defined by $f(x) =$ area of triangle x, is

(A) *one-one* and *into* (B) *one-one* and *onto*
(C) *many-one* and *onto* (D) *many-one* and *into*

28. If one root of a quadratic equation $ax^2 + bx + c = 0$ be equal to the n-th power of the other, then

(A) $(ac)^{\frac{n}{n+1}} + b = 0$
(B) $(ac)^{\frac{n+1}{n}} + b = 0$
(C) $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$
(D) $(ac^{\frac{1}{n+1}})^n + (a^{\frac{1}{n+1}}c)^{n+1} + b = 0.$

29. The condition that ensures that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in H.P. is

(A) $r^2 - 9pqr + q^3 = 0$
(B) $27r^2 - 9pqr + 3q^3 = 0$
(C) $3r^3 - 27pqr - 9q^3 = 0$
(D) $27r^2 - 9pqr + 2q^3 = 0.$

30. Let p, q, r, s be real numbers such that $pr = 2(q + s)$. Consider the equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$. Then

(A) at least one of the equations has real roots.
(B) both these equations have real roots.
(C) neither of these equations have real roots.
(D) given data is not sufficient to arrive at any conclusion.

31. Let A be an $n \times n$ matrix such that $|A^2| = 1$. Here $|A|$ stands for determinant of matrix A. Then

(A) $|A| = 1$
(B) $|A| = 0$ or 1
(C) $|A| = -1$, 0 or 1
(D) $|A| = -1$ or 1.

32. The set of vectors constituting an orthogonal basis in $\mathbb{R}^3$ is

(A) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

(B) $\left\{ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

(C) $\left\{ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \right\}$

(D) None of these.

33. Suppose A and B are orthogonal $n \times n$ matrices. Which of the following is also an orthogonal matrix? Assume that O is the null matrix of order $n \times n$ and I is the identity matrix of order n.

(A) $AB - BA$

(B) $\begin{pmatrix} A & O \\ O & B \end{pmatrix}$

(C) $\begin{pmatrix} A & I \\ I & B \end{pmatrix}$

(D) $A^2 - B^2$.

34. Let A_{ij} denote the minors of an $n \times n$ matrix A. What is the relationship between $|A_{ij}|$ and $|A_{ji}|$?

(A) They are always equal.

(B) $|A_{ij}| = -|A_{ji}|$ if $i \neq j$.

(C) They are equal if A is a symmetric matrix.

(D) If $|A_{ij}| = 0$ then $|A_{ji}| = 0$.

35. Let A, B and C be three non empty sets. Consider the two relations given below:

$$A - (B - C) = (A - B) \cup C \tag{1}$$
$$A - (B \cup C) = (A - B) - C \tag{2}$$

(A) Both (1) and (2) are correct.
(B) (1) is correct but (2) is not.
(C) (2) is correct but (1) is not.
(D) Both (1) and (2) are incorrect.

36. Suppose X and Y are finite sets, each with cardinality n. The number of bijective functions from X to Y is

(A) n^n (B) $n \log_2 n$ (C) n^2 (D) $n!$.

37. Suppose $f_\alpha : [0,1] \to [0,1], \ -1 < \alpha < \infty$ is given by

$$f_\alpha(x) = \frac{(\alpha+1)x}{\alpha x + 1}.$$

Then f_α is

(A) A bijective (*one-one* and *onto*) function.
(B) A surjective (*onto*) function.
(C) An injective (*one-one*) function.
(D) We can not conclude about the type.

38. The length of the chord on the straight line $3x - 4y + 5 = 0$ intercepted by the circle passing through the points $(1,2)$, $(3,-4)$ and $(5,6)$ is

(A) 12 (B) 14 (C) 16 (D) 18.

39. The medians AD and BE of the triangle with vertices $A(0, b)$, $B(0, 0)$ and $C(a, 0)$ are mutually perpendicular if

(A) $b = \sqrt{2}a$ (B) $a = \pm\sqrt{2}b$ (C) $b = -\sqrt{2}a$ (D) $b = a$.

40. The equations $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta + b\cos\theta$, $(0 \leq \theta \leq 2\pi$ and a, b are arbitrary constants) represent

(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola.

41. A straight line touches the circle $x^2 + y^2 = 2a^2$ and also the parabola $y^2 = 8ax$. Then the equation of the straight line is

(A) $y = \pm x$ (B) $y = \pm(x + a)$
(C) $y = \pm(x + 2a)$ (D) $y = \pm(x - 21)$.

42. In an ellipse, the distance between its foci is 6 and its minor axis is 8. Then its eccentricity is

(A) $\frac{4}{5}$ (B) $\frac{1}{\sqrt{52}}$ (C) $\frac{3}{5}$ (D) $\frac{1}{2}$.

43. Four tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the ends of its latera recta. The area of the quadrilateral so formed is

(A) 27 (B) $\frac{13}{2}$ (C) $\frac{15}{4}$ (D) 45.

44. If the distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$, then the equation of the hyperbola is

(A) $y^2 - x^2 = 32$ (B) $x^2 - y^2 = 16$
(C) $y^2 - x^2 = 16$ (D) $x^2 - y^2 = 32$.

45. The value of $\lim_{x \to 0} \dfrac{\tan^2 x - x \tan x}{\sin x}$ is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) None of these.

46. Let $I = \int (\sin x - \cos x)(\sin x + \cos x)^3 dx$ and K be a constant of integration. Then the value of I is

(A) $(\sin x + \cos x)^4 + K$
(B) $(\sin x + \cos x)^2 + K$
(C) $-\frac{1}{4}(\sin x + \cos x)^4 + K$
(D) None of these.

47. The Taylor series expansion of $f(x) = \ln(1 + x^2)$ about $x = 0$ is

(A) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n}$
(B) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}$
(C) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n+1}}{n+1}$
(D) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n+1}$.

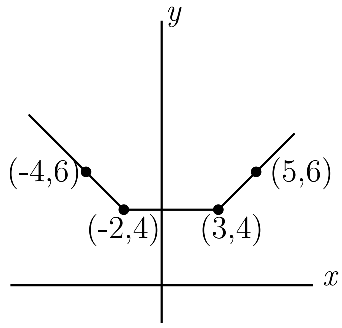
48. $\lim_{x \to 1} \dfrac{x^{\frac{1}{3}} - 1}{x^{\frac{1}{4}} - 1}$ equals

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) 1 (D) None of these.

49. The domain of the function $\ln(3x^2 - 4x + 5)$ is

(A) set of positive real numbers
(B) set of real numbers
(C) set of negative real numbers
(D) set of real numbers larger than 5.

50. The piecewise linear function for the following graph is



(A) $f(x) = \begin{cases} = x, x \leq -2 \\ = 4, -2 < x < 3 \\ = x+1, x \geq 3 \end{cases}$

(B) $f(x) = \begin{cases} = x-2, x \leq -2 \\ = 4, -2 < x < 3 \\ = x-1, x \geq 3 \end{cases}$

(C) $f(x) = \begin{cases} = 2x, x \leq -2 \\ = x, -2 < x < 3 \\ = x+1, x \geq 3 \end{cases}$

(D) $f(x) = \begin{cases} = 2-x, x \leq -2 \\ = 4, -2 < x < 3 \\ = x+1, x \geq 3 \end{cases}$

51. The area bounded by $y = x^2 - 4, y = 0$ and $x = 4$ is

(A) $\frac{64}{3}$ (B) 6 (C) $\frac{16}{3}$ (D) $\frac{32}{3}$.

52. $\lim\limits_{x \to -1} \dfrac{1+\sqrt[3]{x}}{1+\sqrt[5]{x}}$ equals

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 1 (D) ∞.

53. Four squares of sides x cm each are cut off from the four corners of a square metal sheet having side 100 cm. The residual sheet is then folded into an open box which is then filled with a liquid costing Rs. x^2 per cm^3. The value of x for which the cost of filling the box completely with the liquid is maximized, is

(A) 100 (B) 50 (C) 30 (D) 10.

54. $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$ equals

(A) -1 (B) 0 (C) 1 (D) Does not exist.

55. $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ equals

(A) -1 (B) 0 (C) 1 (D) Does not exist.

56. $\lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$ equals

(A) -1 (B) 0 (C) 1 (D) Does not exist.

57. Let $y = \lfloor x \rfloor$, where $\lfloor x \rfloor$ is greatest integer less than or equal to x. Then

(A) y is continuous and *many-one*.
(B) y is not differentiable and *many-one*.
(C) y is not differentiable.
(D) y is differentiable and *many-one*.

58. $\lim_{x \to 1} \dfrac{x^{16} - 1}{|x - 1|}$ equals

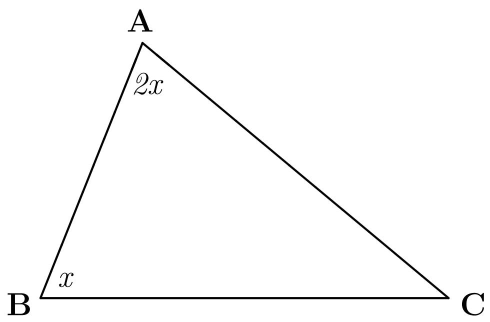
(A) -1 (B) 0 (C) 1 (D) Does not exist.

59. If in a $\triangle ABC, \angle B = \frac{2\pi}{3}$, then $\cos A + \cos C$ lies in

(A) $[\ -\sqrt{3}, \sqrt{3}\ ]$ (B) $(\ -\sqrt{3}, \sqrt{3}\ ]$

(C) $(\ \frac{3}{2}, \sqrt{3}\ )$ (D) $(\ \frac{3}{2}, \sqrt{3}\ ]$.

60. Which of the following relations is true for the following figure?



(A) $b^2 = c(c + a)$ (B) $c^2 = a(a + b)$

(C) $a^2 = b(b + c)$ (D) All of these.

61. The value of $\sin^6 \frac{\pi}{81} + \cos^6 \frac{\pi}{81} - 1 + 3\sin^2 \frac{\pi}{81} \cos^2 \frac{\pi}{81}$ is

(A) $\tan^6 \frac{\pi}{81}$ (B) 0 (C) -1 (D) None of these.

62. The number of values of x for which the equation $\cos x = \sqrt{\sin x} - \frac{1}{\sqrt{\sin x}}$ is satisfied, is

(A) 1 (B) 2 (C) 3 (D) more than 3.

63. If $\sin^{-1}\frac{1}{\sqrt{5}}$ and $\cos^{-1}\frac{3}{\sqrt{10}}$ lie in $\left[0, \frac{\pi}{2}\right]$, their sum is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\sin^{-1}\frac{1}{\sqrt{50}}$ (D) $\frac{\pi}{4}$.

64. If $\cos 2\theta = \sqrt{2}(\cos\theta - \sin\theta)$ then $\tan\theta$ equals

(A) 1 (B) 1 or -1 (C) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$ or 1 (D) None of these.

65. The value of $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \cdots + \sin^2 90^\circ$ is

(A) 8 (B) 9 (C) 9.5 (D) None of these.

66. If $\sin(\sin^{-1}\frac{2}{5} + \cos^{-1} x) = 1$, then x equals

(A) 1 (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) None of these.

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